

## Information sheet.

### Exposing errors and misconceptions

Learners make mistakes for many reasons. They may just be due to lapses in concentration, hasty reasoning, or a failure to notice important features of a problem. Others, however are symptoms of more profound difficulties.

There is now a vast body of research literature documenting learners' mistakes in mathematics (e.g. [1, 2]). This work shows that mistakes are often the result of consistent, alternative interpretations of mathematical ideas. These should not be dismissed as 'wrong thinking' as they are natural, sometimes necessary, stages of conceptual development. For example, most learners generalise from their early experiences that:

- "you can't divide smaller numbers by larger ones";
- "division always makes numbers smaller";
- "the more digits a number has, then the larger is its value";
- "shapes with bigger areas have bigger perimeters";
- "letters represent particular numbers";
- "'equals' means 'makes'".

There are two common ways of reacting to these misconceptions.

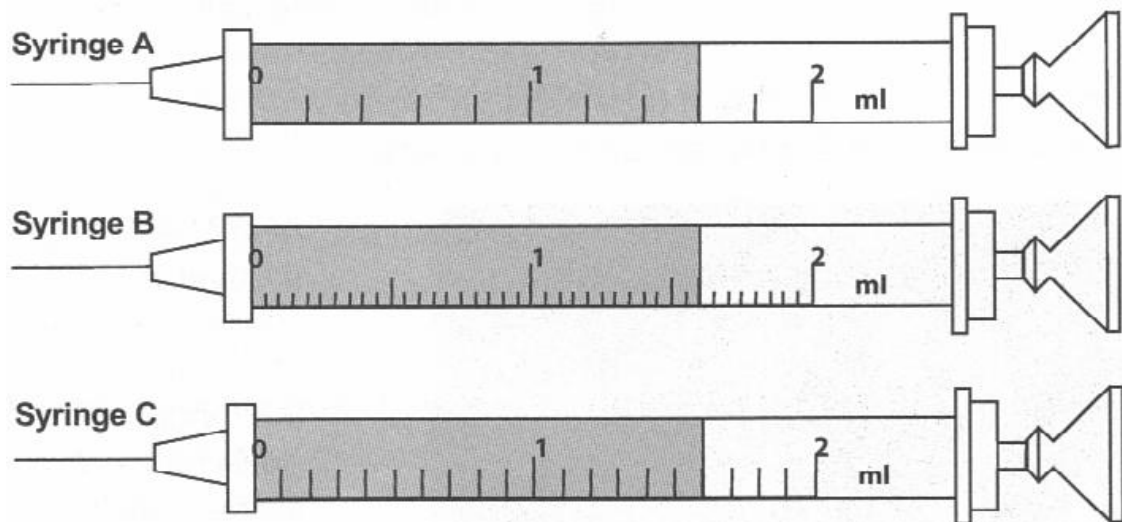
- Try to avoid them:  
"If I warn learners about the misconceptions as I teach, they are less likely to happen. Prevention is better than cure."
- Provoke them and use them as learning opportunities:  
"I actively encourage learners to make mistakes and learn from them".

The first reaction invokes a medical metaphor which is unhelpful. Misconceptions are not diseases that can be avoided by 'better teaching', neither are they 'caught' through a casual encounter. They are reasoned, alternative ways of thinking. **Research suggests that teaching approaches which encourage the exploration of misconceptions through discussions result in deeper, longer term learning than approaches which try to avoid mistakes by explaining the 'right way' to see things from the start [3, 4, 5].** The research studies have taught us the following.

- It is helpful if discussions focus on known difficulties. Rather than posing many questions in one session, it is better to focus on a challenging question

and encourage a variety of interpretations to emerge, so that learners can compare and evaluate their ideas.

- Questions can be juxtaposed in ways that create a tension (sometimes called a 'cognitive conflict') that needs resolving. Contradictions arising from conflicting methods or opinions can create an awareness that something needs to be learned. For example, asking learners to say how much medicine is in each of syringes A-C (below) may result in answers such as "1.3 ml, 1.12 ml and 1.6 ml". "But these quantities are all the same." This provides a start for a useful discussion on the denary nature of decimal notation.



- Activities should provide opportunities for meaningful feedback. This does not mean providing summative information, such as the number of correct or incorrect answers. More helpful feedback is provided when learners compare results obtained from alternative methods until they realise why they get different answers.
- Sessions should include time for whole group discussion in which new ideas and concepts are allowed to emerge. This requires sensitivity so that learners are encouraged to share tentative ideas in a non-threatening environment.
- Opportunities should be provided for learners to 'consolidate' what has been learned through the application of the newly constructed concept.

There are several ways of creating the conditions in which learners can feel able to discuss common mistakes and misconceptions without feeling threatened. Two suggestions are given below.

- Ask learners to solve a problem in pairs. Collect in some examples of different responses and write these on the board anonymously. Add a few extra ones

that illustrate other interesting discussion points that you have already prepared. Then ask learners to discuss and debate these responses.

- Give learners a completed past examination paper to mark. This should illustrate interesting and significant errors that show underlying misconceptions. It is often good to include responses that the learners themselves have made, but rewritten so that they become anonymous. Learners usually seem to enjoy the role shift when they are asked to become examiners. Encourage them to write comments indicating the source of the error and helpful advice to the 'candidate'.

#### References.

1. Higgins S., Ryan J., Swan M. and William J., ' Learning from mistakes, misunderstandings and misconceptions in mathematics', in Thompson I. (ed.), *National numeracy and key stage 3 strategies*, London 2002, DfES.
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3. Bell A., 'Some experiments in diagnostic teaching', *Educational Studies in Mathematics*, vol.24,no.1,1993,pp115-117.
4. Bell A., Swan M., Onslow B., Pratt K and Purdy D., *Diagnostic teaching for long term learning*. Report of ESRC project HR 8491/1, 1985, Shell Centre for Mathematical education, University of Nottingham.
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